The supply chain complexity triangle

Uncertainty generation in the supply chain

Richard Wilding
Cranfield Centre for Logistics and Transportation, Cranfield School of Management, Cranfield, Bedford, UK

Received December 1996
Revised September 1998

Introduction

Today's market place is increasingly dynamic and volatile. Globalisation is resulting in many organisations experiencing market pressures that are forcing a fundamental rethink of the way business is conducted. Trade-offs between for example labour costs, transportation costs, inventory costs and response time to customer are becoming increasingly complex (Sharma, 1997). It is no longer seen as possible only to focus on one's individual organisation to gain competitive advantage. It is now recognised that the success of the individual organisation is dependent on the performance and reliability of its suppliers and also customers.

Christopher (1992) emphasises this by stating:

> Competition in the future will not be between individual organisations but between competing supply chains.

One key issue known to impact on the effectiveness of a supply chain is that of uncertainty (Davis, 1993). Uncertainties in supply and demand are recognised to have a major impact on the performance of the manufacturing function. Research at Intel (Oliver and Houlihan, 1986), investigating the match between actual call off and the actual forecast, estimated that supply and demand were in equilibrium for 35 minutes in ten years!

The "supply chain complexity triangle" provides an explanation for this far-from-equilibrium behaviour and gives a useful insight into the generation of uncertainty within supply chains (Wilding, 1997b). Three interacting yet independent effects would seem to cause the dynamic behaviour experienced within supply chains. These are deterministic chaos, parallel interactions and demand amplification. The combination of these effects can significantly increase the degree of uncertainty within a supply chain system. Figure 1 depicts these three effects and their interactions. The paper will describe each effect in turn before discussing the implications for supply chain strategy and manufacturing logistics.

Deterministic chaos in supply chains

The Collins English dictionary describes chaos as meaning "complete disorder and confusion". However, within this paper the term chaos describes

International Journal of Physical Distribution & Logistics Management, Vol. 28 No. 8, 1998, p. 599-616. © MCB University Press, 0960-0035

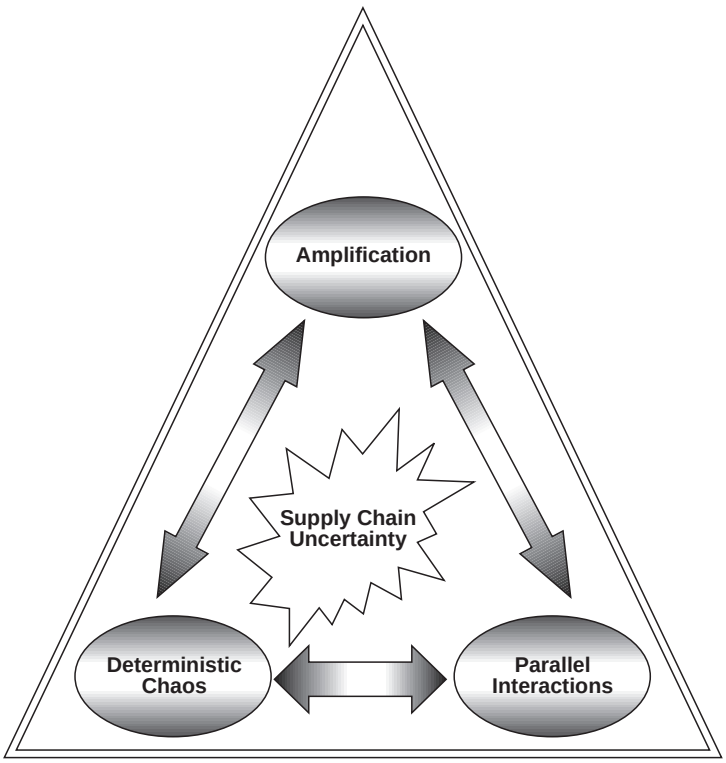

Figure 1.
The supply chain complexity triangle

deterministic chaos. The definition used in this work is adapted from that proposed by Kaplan and Glass (1995, p. 27) and Abarbanel (1996, p. 15):

Chaos is defined as aperiodic, bounded dynamics in a deterministic system with sensitivity dependence on initial conditions, and has structure in phase space.

The key terms can be defined as follows:

- *Aperiodic*: the same state is never repeated twice.
- *Bounded*: on successive iterations the state stays in a finite range and does not approach plus or minus infinity.
- *Deterministic*: there is a definite rule with no random terms governing the dynamics.
- *Sensitivity to initial conditions*: two points that are initially close will drift apart as time proceeds.
- *Structure in phase space*: Non-linear systems are described by multidimensional vectors. The space in which these vectors lie is called phase space (or state space). The dimension of phase space is an integer (Abarbanel, 1996). Chaotic systems display discernible patterns when viewed. Stacey (1993a, p.228) emphasises this by defining chaos as:

  order (a pattern) within disorder (random behaviour).

Professor Ian Stewart proposes the following simplified definition (Stewart, 1989, p. 17):

> Stochastic behaviour occurring in a deterministic system.

Stochastic means random or lawless, deterministic systems are governed by exact unbreakable laws or rules. So chaos is "Random (or lawless) behaviour governed entirely by laws!"

Chaos is deterministic, generated by fixed rules that in themselves involve no element of chance (hence the term deterministic chaos). In theory, therefore, the system is predictable, but in practice the non-linear effects of many causes make the system less predictable. The system is also extremely sensitive to the initial conditions, so an infinitesimal change to a system variable's initial condition may result in a completely different response. One implication of chaos theory is that random behaviour may be more predictable than was originally thought. Information collected in the past, and subsequently dismissed as being too complicated, may now be explained in terms of simple rules. The complication is that due to the nature of chaotic systems there are fundamental limits to the horizon and accuracy of prediction. Past patterns of system behaviour are never repeated exactly, but may recur within certain limits.

The Appendix outlines a simple experiment to demonstrate some of the characteristics of a chaotic system using a spreadsheet and discusses the impact of chaotic systems on computer accuracy.)

Chaos resulting from supply chain decision-making processes

The "beer game", a management game developed some three decades ago to introduce students and industrialists to the concepts of economic dynamics and management decision making has shed further light on the dynamic behaviour of supply chains. The game shows how the inter-relating feedback loops within the supply chain give rise to complex behaviour within what seems to be a very simple business system. The game is run with four teams of participants; each team is a company within the supply chain i.e. a retailer, wholesaler, distributor and factory. A team of researchers based at MIT investigating managerial decision-making behaviour found that participants apply simple rules for making ordering decisions when playing the game (Larsen *et al.*, 1989). It has been found by the analysis of many runs of the beer game that participants vary slightly in the application of the rules. For example some participants take into account all the inventory in the supply line while others ignore it altogether or forget it occasionally, participants may have a slow response to inventory fluctuations away from their desired level while others may respond fast and try to achieve their target more aggressively. It has been subsequently possible to analyse and simulate the decision rules made to find which rules are the most effective. It was recognised that generally simulations were run over a short period of time, say 60 weeks. This time period is less than the fundamental period of the system and therefore will not reveal the existence of complex modes of behaviour within the system.

It has been found that within the simple model outlined above, one in four management teams in the supply chain create deterministic chaos in the ordering patterns and inventory levels. This produces costs to the system that are considerably sub-optimal, exceeding the minimum possible costs by over 500 per cent (Mosekilde *et al.*, 1991). The results also showed that the slightest change in policy could result in a stable output flipping into the chaotic region, i.e. a transcription error when inputting an order, an order delayed in the post, a manager forgetting something or inputting it a day late, all these everyday seemingly inconsequential delays or errors can have a dramatic and costly effect on the management of the supply chain.

The authors demonstrated that the more complex forms of chaos occur when an aggressive stock adjustment policy with low desired inventory and the tendency to neglect supply line adjustments is applied. When managers are over ambitious in setting low target inventory levels, chaos is more likely to occur and generally costs are likely to rise. This argument is witnessed in practical industrial environments: driving inventory down to low levels can result in distress due to stockouts, rapid and erratic re-ordering and poor customer service levels.

Increasingly within industry, managerial decision-making rules are being formalised by computer algorithms. A conclusion that can be drawn is that if such algorithms are inappropriately designed, chaotic behaviour can be generated, thus contributing to the uncertainty experienced in the supply chain.

Chaos resulting from supply chain control systems

Research undertaken by Wilding (1997b) to gain an insight into the potential generation of chaos within warehouse supply chains also provides evidence that uncertainty can be generated by deterministic chaos. Figure 2 depicts examples of the type of supply chain structures used in the investigation. The supply chains investigated are characterised by automatic inventory control algorithms and EDI (electronic data interchange) between the echelons. It is accepted that warehouse supply chains are in reality more complex, but the model does capture the main components of such a system.

The simulation approach used in this research was a development of that created by Mike Wilson at Logistics Simulation Ltd. This software and approach to the simulation was chosen as it is used commercially as a training and strategic development tool by a number of blue chip companies including ICI and Black & Decker (Wilson, 1994). It has been tested widely in industry and is shown to mimic with good accuracy the characteristics of actual warehouse supply chains. The simulation has been subjected to rigorous validation by the author, engineers and scientists within the University of Warwick and also external practitioners and academics.

The investigations used an automatic re-order algorithm within the warehouse, which forecasts demand, calculates the optimum inventory cover level and places an order to account for expected demand for a given period.

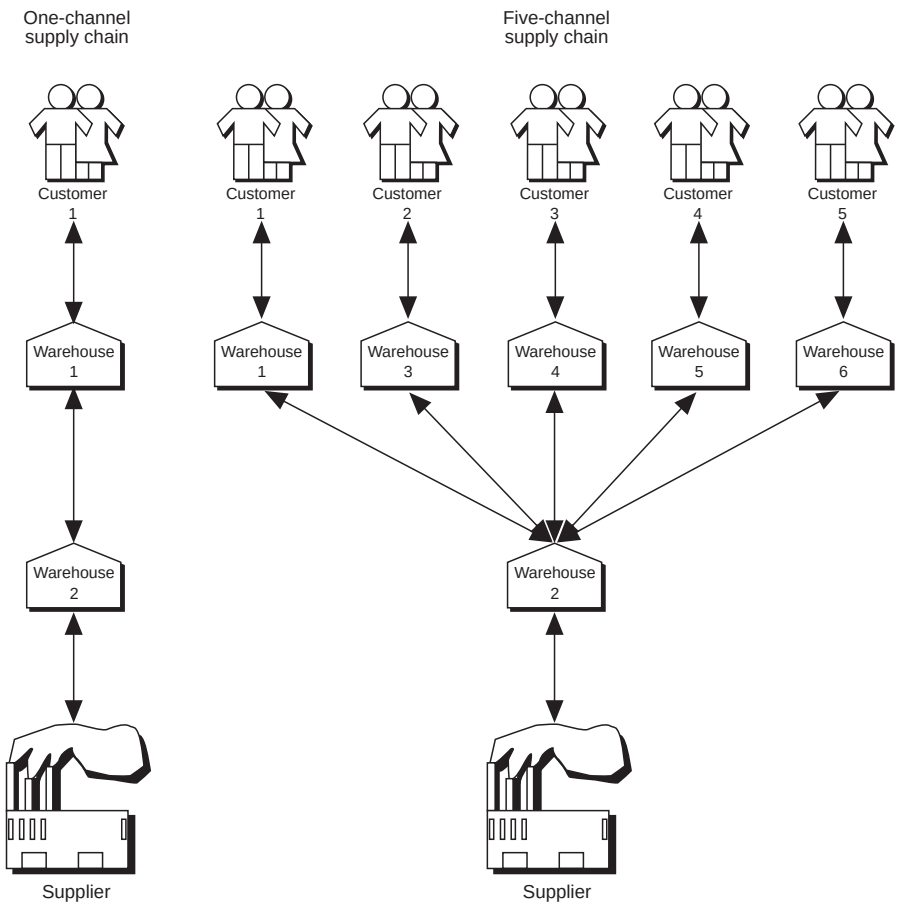

Figure 2. Examples of supply chain structures investigated

This is a widely used re-order and stock control algorithm used in industry (Silver and Peterson, 1985, pp. 105-7; Waters, 1992).

The main emphasis of the investigation is to quantify how the increasing complexity of the supply chain resulting from increasing the number of echelons and/or channels impacts on the degree of chaos. This is measured by the Lyapunov exponent value that was then used to calculate the average prediction horizon of the data from each warehouse (Wilding, 1997a).

The investigations demonstrated that warehouse supply chains acted as characteristic chaotic systems exhibiting sensitivity to initial conditions, "islands of stability" (i.e. under certain conditions the supply chain did not generate chaos), characteristic patterns, the reductionist view was invalidated, and finally, the chaos undermined computer accuracy (Wilding, 1998).

"Chaotic spikes" in demand were also generated by the supply chains investigated (Wilding, 1998). A "chaotic spike" is a rapid change in demand generated internally by the system's chaotic nature and is not caused by any

external event. Unexpected "chaotic spikes" have also been witnessed in a spreadsheet model produced by Levy (1994) of a simple supply chain. Levy concludes that, within the chaotic system, dramatic change can occur unexpectedly. Small external changes can occur causing large changes in demand and inventory.

Parallel interactions in supply chains

Serial interactions in supply chains occur between each echelon in the supply chain i.e. a single customer and a supplier. An example of a serial interaction would be demand amplification (Forrester, 1961). The term "parallel interaction" has been defined to describe interactions that occur between different channels of the same tier in a supply network. An example of parallel interactions occurs when a first tier supplier cannot supply a customer; this results in re-scheduling within the customer organisation resulting in the customer changing its requirements from other first tier suppliers. This results in uncertainty being generated within the supply network. The supplier is affected by an occurrence in a parallel supply chain, which at first would seem unrelated. Figure 3 shows a simplified diagrammatic representation of these effects.

Parallel interactions in an automotive supply chain

"Parallel interactions" within the supply chain were observed by Jones (1990, p. 291) in an automotive supply chain; however no quantitative analysis of this phenomenon was undertaken.

Jones noticed that poor delivery or quality performance from some suppliers in the network affects the efficiency of the good (often just-in-time (JIT)) suppliers. Jones (1990) suggests that the good suppliers face schedule "ripple" variations caused by the poor suppliers. The supply chain structure investigated by Jones (1990) forms the basis of an investigation undertaken by Wilding (1997b).

The model developed by Wilding (1997b) represents a simple supply network of four suppliers producing sub-assemblies that are combined by the customer into a finished product. The structure is based on a detailed model developed by Jones (1990). This model was developed to investigate logistics performance within an automotive supply chain.

The investigation demonstrated that "parallel interactions" between suppliers within a supply network do occur. The impact of the interactions on individual suppliers and the assembler has been quantified by calculating the percentage of time the company or assembler would be stopped due to the interactions. In practice, an actual stoppage may not occur but organisations may be forced to re-schedule thus resulting in fluctuations and uncertainties in demand being experienced by suppliers. The "parallel interactions" within the network can be reduced by buffering with inventory; however even for large buffers, interactions do occur but less frequently.

The investigation also demonstrates and quantifies the impact of variability between the forecast demand and the actual demand. Increased variability between the forecast and actual demand results in both suppliers and the assembler experiencing increased stoppages due to interactions.

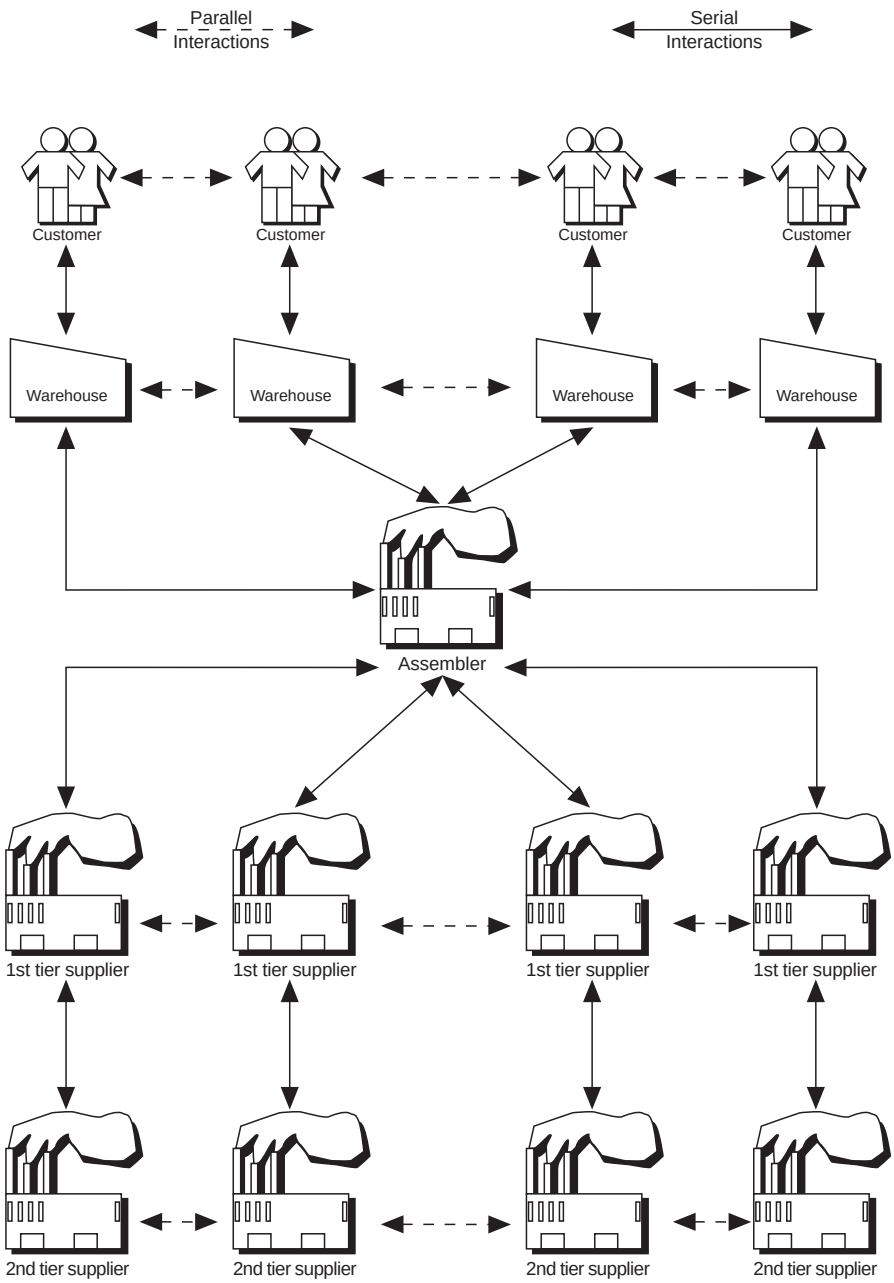

Figure 3. Serial and parallel interactions in a supply network

This work also highlights that a JIT supplier within a supply network is susceptible to interactions from "rogue" suppliers (i.e. poor quality and delivery performance suppliers) that can dramatically impact on the JIT suppliers' utilisation. Inventory is required to buffer the JIT supplier from such interactions,

which may in some situations remove the benefits of operating "just-in-time". A "rogue" supplier within a supply network does not only affect the assembler but also other suppliers in the network. This further emphasises the need for a holistic approach to supply chain management recognising that the supply network must be treated as a system and not a collection of individual companies.

Parallel interactions can also be a significant source of uncertainty within the supply chain. Even with large inventory buffers, the parallel interactions have some impact on the utilisation of both the assembler and other suppliers in the network. The results from this investigation demonstrates that upwards of 18 per cent of the time suppliers and the assembler can be stopped by parallel interactions or their programmes can be disrupted, thus forcing the assembler and/or supplier to re-schedule production.

Demand amplification in supply chains

The first piece of work undertaken to understand the dynamic behaviour of simple linear supply chains was carried out by Jay Forrester of MIT (Forrester, 1961). One of the key outputs of Forrester's (1961)work is a practical demonstration of how various types of business policy create disturbances which are often blamed on conditions outside the system. Random, meaningless sales fluctuations can be converted by the system into apparently annual or seasonal production cycles thus sub-optimising the use of capacity and generating swings in inventory. A change in demand is amplified as it passes between organisations in the supply chain.

This type of amplification behaviour has been summarised as the "Forrester flywheel effect" (Houlihan, 1987). Figure 4 shows the nature of this relationship.

Forrester's work has been further developed by Towill (1996) and Towill and Naim (1993). Towill has investigated ways of reducing demand amplification and has demonstrated the impact of current supply chain strategies such as JIT, vendor integration and time-based management on reducing the amplification effect.

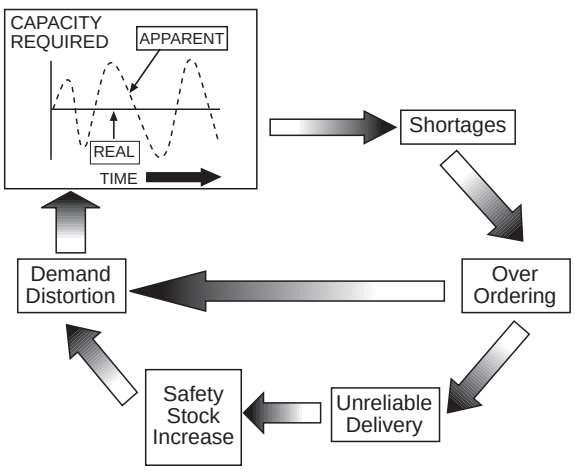

Figure 4.
The Forrester flywheel effect (adapted from Houlihan, 1987)

More recently, Lee *et al.* (1997a, 1997b) describe the "bullwhip" effect occurring in supply chains. The bullwhip effect is the term used by Procter and Gamble to describe the amplification and demand distortion that occurs within the supply chain. The authors refer to four causes of the bullwhip effect:

- Demand forecast updating – amplification due to increasing safety stock and stock in the pipeline.
- Order batching – customers tend to order goods at certain times during the week, for example Monday morning. Organisations running materials requirements planning or distribution requirements planning to generate purchase orders do so at the end of the month. These periodic batching of processes result in surges in demand at certain points in time.
- Price fluctuations – the impact of promotion results in forward buying, this occurs particularly in the grocery industries. For example, supermarkets in the United Kingdom recently reduced the price of baked beans to three pence per tin. This resulted in customers buying large quantities of the product; however it is unlikely that the price will result in increased consumption of the product. As a result the customer's consumption pattern does not reflect the buying pattern. This results in bigger variations in demand patterns.
- Rationing and shortage gaming – when product demand exceeds supply organisations often ration sales to retail customers. This results in end customers placing multiple orders with different retailers hoping this will result in more chance of the product being received within a given lead-time. This of course results in excess demands for products and the manufacturing organisation increasing capacity to satisfy all the apparent orders.

Their investigation is very much analogous to Forrester's (1961) original investigation into amplification within the supply chain. However, Lee *et al.* (1997a, 1997b) have taken the original concepts and used examples of relevance to today's market conditions.

Discussion

The "supply chain complexity triangle" results because each source of uncertainty can act as a stimulus for one of the other sources of behaviour to occur. For example, demand amplification may result in a system operating initially in an "island of stability" to be pushed into a chaotic mode of operation. If the system is operating in a chaotic mode of operation, the occurrence of a "chaotic spike" within one echelon may result in demand amplification occurring in the echelons down stream. If, due to the demand amplification and chaos, capacity is exceeded in one supply channel the resulting mis-supply may cause parallel interactions which in turn may result in amplification and chaos. The three interacting phenomena therefore result in complex demand patterns with limited forecast horizons. This uncertainty will result in additional costs being experienced by those in the supply chain.

A further paradox identified about the "supply chain complexity triangle" is that methods to reduce the magnitude of one effect may result in an increase in magnitude in one of the other sources of uncertainty. This was witnessed in an investigation where the supplier lead-time was reduced, on the basis this is known to reduce the degree of amplification generated within supply chains (Wikner *et al*., 1991). However, the reduction of the lead-time resulted in an increase in the degree of chaos and hence a reduction of the prediction horizon of the data series. This result also confirms the finding of Gordon and Greenspan (1994) who recognised that, for chaotic environments, increasing the time interval between actions moved the system towards stability, therefore the increased supplier lead-time resulted in increased stability i.e. a reduction in chaos. This therefore results in a trade-off between amplification and chaos.

Wilding (1997b) demonstrates that parallel interactions can be buffered with increased inventory within the supply chain. However, research undertaken into demand amplification demonstrates that increasing the amount of inventory cover results in increased amplification (Wikner *et al*., 1991). This trade-off also needs to be recognised.

Implications for supply chain strategy

The conclusion that complex forms of behaviour can be generated within supply chains results in the requirement to refocus the ways supply chains are strategically managed. The conventional view that supply chain success is dependent on stability and consensus is challenged.

The complexity experienced in the supply chain can be viewed as a threat and something that needs to be avoided and/or reduced; however achieving these objectives may be difficult in practice. An alternative view is presented by authors such as Parker (1994), Stacey (1993b) and McMaster (1996) who argue that the complexity experienced may force organisations to innovate and learn. If everything were stable, organisations would not need to develop new structures or patterns of behaviour. Over time, this would lead to lack of innovation and subsequent loss of competitive advantage.

By understanding the trade-offs within the "supply chain complexity triangle" organisations could potentially improve the quality of service to customers by ensuring improved availability of goods, and also reduce costs within the system by more effective management of inventory and resources. This therefore improves both cost advantage and value advantage for the organisations.

The analysis further emphasises the importance of treating the supply chain as a complete system. The whole is not the sum of the parts. Small changes made to optimise one echelon of the supply chain can result in massive changes in other parts of the supply chain. This may subsequently result in the sub-optimisation of the total system performance.

Long term planning within chaotic systems is also particularly difficult. Small disturbances are multiplied over time and because of the non-linear relationships present, the system is very sensitive to initial conditions. Traditional materials requirements planning (MRP) systems used in industry

are reliant on long-term sales forecasts which are usually inaccurate. This can result in excessive stock holding (Burbidge, 1983).

Tom McGuffog (1997) of the international organisation Nestlé recently concluded that the complex statistical forecasting packages employed by their organisation do not substantially assist the interpretation of demand. He observes that for these systems to be successful there would need to be patterns susceptible to statistical analysis and prognosis. These simple patterns are not observed in practice, and traditional forecasting techniques have had very limited success. These observations provide further evidence that the complex dynamics generated may be chaotic in nature.

The benefit of allocating resources to more and more complex models for forecasting may be small. Short-term forecasts and prediction of patterns can be made with reasonable accuracy. Chaotic systems trace repetitive patterns that may make it possible to forecast levels of stock or demand within certain tolerance bands.

Non-linear dynamic analysis can also be used to estimate the forecast horizon of supply chain systems. This has the benefit of focusing resources on forecasting up to that horizon and not wasting resources on trying to forecast past this horizon into the unpredictable future. The use of Lyapunov exponents and the subsequent calculation of the prediction horizon can be used as a technique for quantifying what "short term" and "long term" mean within a business environment. Short-term management and strategies can be defined for operation within the prediction horizon. Long-term policies and strategies are defined as those that function outside this forecast horizon.

The concept of short-term strategic management may be the most effective strategic approach for management within supply chains (Saisse and Wilding, 1997). Managers within an organisation need to be aware of the strategic consequences of their daily short-term decisions. These decisions must be aligned with the overall business strategy of the organisation, and this raises the requirement for management tools and techniques. This type of approach to management within uncertain environments has the potential to be applied across the complete supply chain (Saisse and Wilding, 1997).

A further implication of this work applies to the evolving structure of supply chains. Studies of automotive parts supply chains have led to forecasts that by the year 2005 the structure of the supply chain will change dramatically with the requirement for an increase in echelons but a reduction in the number of channels (Disney *et al.*, 1997). The "supply chain complexity triangle" raises a number of key issues about this supply chain re-engineering process. Increasing the number of echelons will result in an increase in the amount of chaos and amplification experienced, but reducing the number of channels will result in a reduction in the number of parallel interactions. The strategists involved in this work would be wise to understand the implications of this trade-off.

Implications for manufacturing logistics

The purpose of inventory control systems as described by Waters (1992, p. 16) is as follows:

Inventory control is based on the use of quantitative models which relate demand, cost and other variables to find optimal values for order quantities, timing of orders and so on.

The implication of supply chains readily generating deterministic chaos is that a system which is meant to control and level fluctuations, and consequently buffer the system from instability, can create dynamics which turn a stable predictable, demand pattern into a demand pattern which is unpredictable with occasional explosive changes in demand, so further destabilising the system. Thus a system designed to optimise stock holding and order management can actually increase unpredictability and costs incurred across the total supply chain.

Manufacturing planning systems are often run in a batch mode at particular time intervals (for example one every four weeks). This is often a result of the time it takes to do all the calculations and processing. One implication of supply chains behaving chaotically is that if the time period between runs of the planning system is greater than the prediction horizon, the planning for events outside the prediction horizon could be completely inaccurate. By running the planning system with a period of less than the prediction horizon, uncertainty due to chaos will be minimised.

However, rather than learning to live with chaos, it may be better to remove it altogether. The key to the removal of chaos is the use of systems that do not have direct feedback loops. The exponentially smoothed forecasting system used in the warehouse model is one such feedback loop. Simulations using simple re-order point systems do not produce chaotic behaviour as no feedback loops are present; however demand amplification has been shown to be a major drawback with this type of system. Many lean approaches to manufacturing do not rely on complex feedback systems. Focusing on the uninterrupted flow of material that matches the pull from the customer, which is the basis of such techniques, can be seen to eliminate feedback and consequently the conditions required to produce further chaos. However, the misapplication of lean manufacturing, such as the wholesale reduction of inventory and lead-times, can result in the system exhibiting increased chaos. Period batch control (PBC) is another technique, which, if used appropriately, can remove chaos. It enables parts to be made in balanced product sets that match customer demand. No production of parts should be made for stock intended to cover future requirements (Burbidge, 1983). Hill (1996) discusses the use of statistical process control (SPC) in monitoring demand from customers. He proposes a system where production is levelled and strategic stocks are used to buffer against uncertainty. SPC is used to quantify the level of risk and calculate the buffer required. This is not altered unless the system is seen to change dramatically. This form of system also relies on pull from customer demand. However inventory is used to strategically buffer fluctuations and thus level production. This would also result in stable demand being passed on to suppliers further down the supply chain.

Inventory can be used to buffer the uncertainty but this may increase the costs for those operating JIT. Organisations implementing JIT therefore need to ensure that their systems are flexible and responsive enough to cope with the increase in uncertainty that may be experienced. This may account for disappointing

improvements experienced by many implementing JIT. If JIT inventory systems are to be employed, all the business and manufacturing systems need to be reviewed to ensure their flexibility and responsiveness to cope with the possibility of increased uncertainty. This review may result in organisations recognising that inventory buffering and the production techniques outlined above and advocated by Burbidge and Hill may be more appropriate.

Conclusions

The "supply chain complexity triangle" provides a useful structure within which to understand the generation of uncertainty with a supply chain. The key implications for management are as follows:

- Dramatic change can occur unexpectedly. Chaotic spikes in demand can occur, generated by the system and not as the result of external events.
- Long-term planning is very difficult. If long-term plans are made they need to be reviewed on a regular basis.
- Supply chains do not reach stable equilibrium; small perturbations will always prevent equilibrium being achieved.
- Short-term forecasts and prediction of patterns can be made. It is better to allocate resource to the development of effective short-term decision-making processes rather than long term.
- Treat the supply chain as a complete system, small changes made to optimise one echelon of the supply chain can result in massive changes in other parts of the supply chain. Driving down inventory and lead-times may not always improve performance. It could result in the system slipping into chaos.
- Remove chaos by focusing on the customer; communicate demand information as far upstream as possible, using simple lean approaches.
- When changing hardware or software platforms, which are critical to an organisation's operation, undertake detailed validation. Computers are prone to chaos.
- Simulation of systems and non-linear dynamic analysis of key outputs should be a mandatory part of any supply chain re-engineering proposal. Search for "islands of stability". Remember that if the model generates chaos the real system with increased complexity may do so.

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Appendix: simple equations, peculiar behaviour

The use of spreadsheets to demonstrate chaos enables an accessible method to demonstrate the nature of such systems (Durkin and Nevills, 1994). Even simple equations can behave chaotically and these can have a dramatic effect on perceived computer accuracy. To demonstrate this phenomena a simple example will be described.

The example demonstrates chaos by iterating a simple equation using a standard spreadsheet package. The simple equation to be iterated is as follows:

$$X_t = KX_{t-1}^2 - 1$$

This simple equation for certain values of K is chaotic.

For example, using a clear spreadsheet in Microsoft Excel, type "0.54321" in cell A1. In cell A2 type the equation "= 2*(A1*A1)-1" (In this example K = 2). Therefore our new value for X_t = $2(0.54321^2)$–1 or "–0.40985". Now copy the equation down to the subsequent cells in column A. This will result in the equation in Cell A3 reading "=2*(A2*A2)-1" and Cell A4 reading "=2*(A3*A3)-1 etc. If you want to plot the data, use the "Chart wizard" to generate a line graph of the data. You can now start experimenting, introduce a small error by typing "0.54321000000001" and see how over time the results diverge due to chaos. Also change the value of K in the equation.

When K is 1 then a stable periodic orbit occurs, the system is attracted to a cycle of 0, –1, 0, –1. The equation is relatively stable up until K = 1.5 producing periodic behaviour or quasi-periodic behaviour. At K = 1.5 chaos occurs and the dynamics become increasingly complicated as K increases. However, at K = 1.74 the system behaves chaotically but at K = 1.76 stable behaviour occurs. This stable behaviour continues until K = 1.81 and then chaotic behaviour reoccurs. Therefore an "island of stability" is present between K = 1.76 to 1.80. At K=2 a more advanced form of chaotic behaviour can be witnessed. Figures A1 and A2 show plots of the data for K = 1.76 (Stable behaviour) and K = 2 (Chaotic behaviour) respectively.

Table AI shows the iteration of this equation when K = 2 on two different spreadsheet and hardware platforms and two starting conditions differing by a tiny amount. It can be seen that after 40 iterations the results start to diverge rapidly. If K is greater than 2 the equation becomes unstable and the solution approaches infinity. (In doing these experiments you may not get precisely the same numbers as those in the table, this is a consequence of the chaotic nature of the equation.)

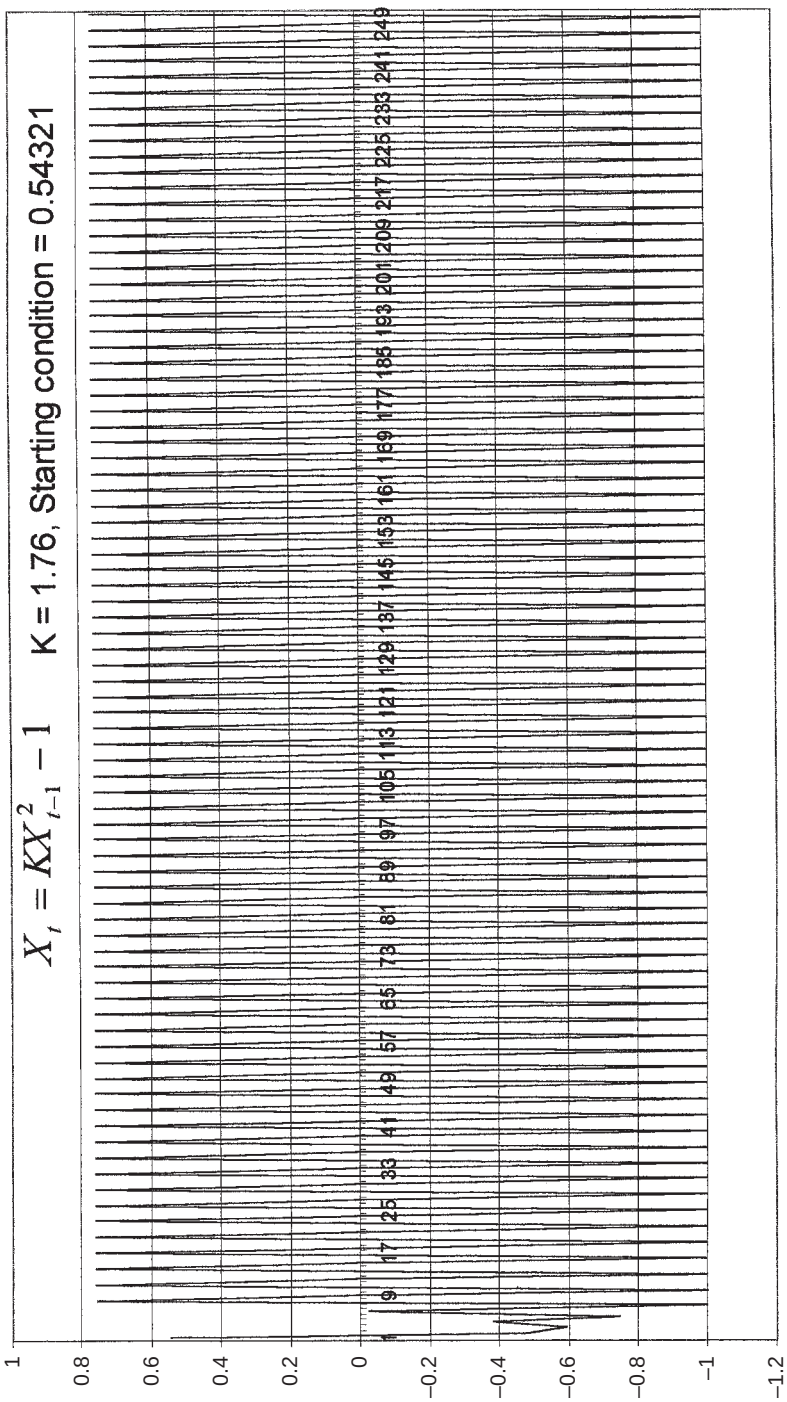

Figure A1.
Graph of the iteration of equation for K = 1.76. Stable behaviour. (Generated with Microsoft Excel)

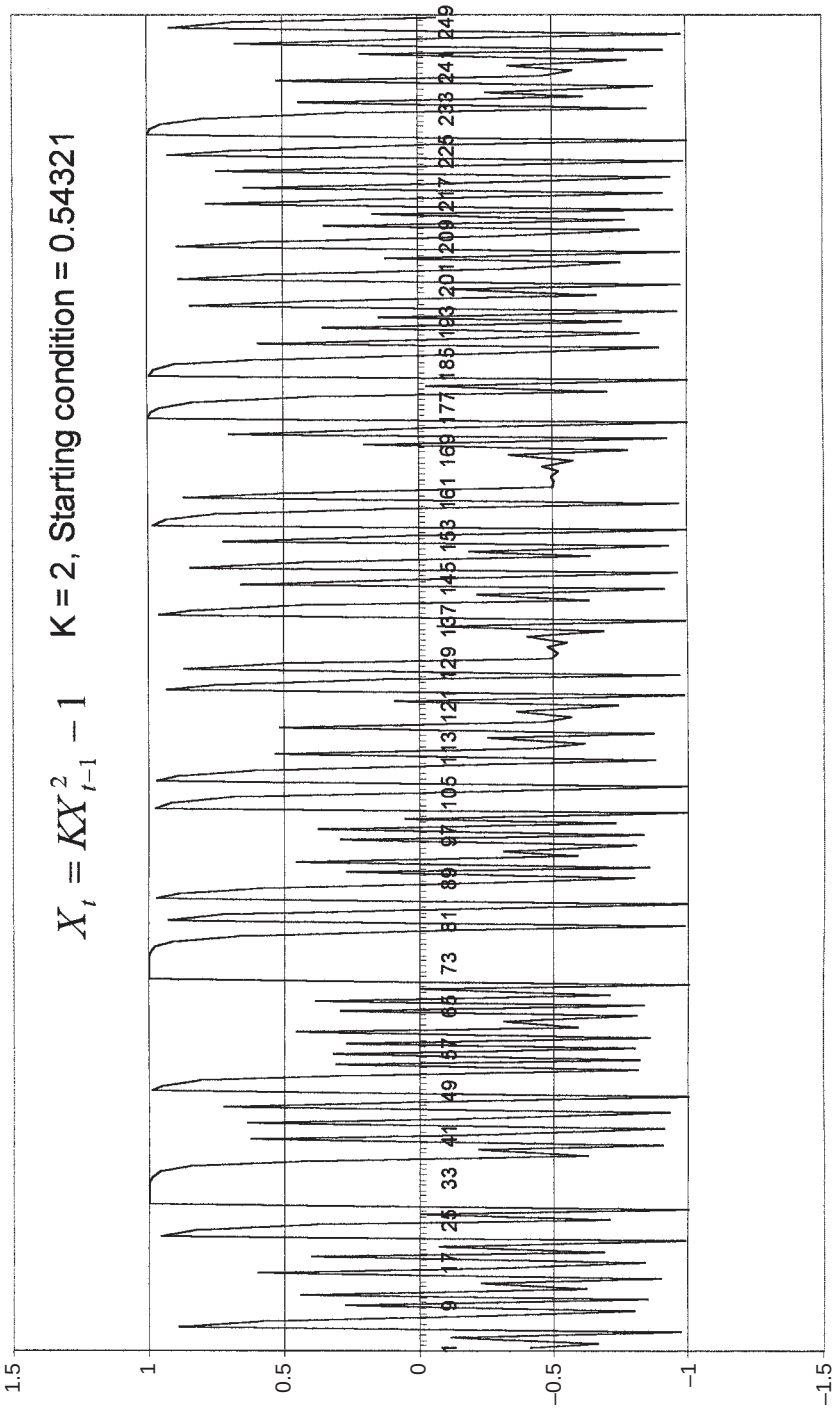

Figure A2. Graph of the iteration of equation for K = 2. Chaotic behaviour. (Generated with Microsoft Excel)

This raises a fundamental issue about the impact of chaos on computer systems. An identical program run on two different makes of computer, or different standard software packages doing the same calculations can produce significantly different results.

Peitgen *et al.* (1992) further emphasises this point by stating:

> More and more massive computations are being performed now using black box software packages developed by sometimes very well known and distinguished centers. These packages, therefore, seem to be very trustworthy, and indeed they are. But this does not exclude the fact that the finest software sometimes produces total garbage, and it is an art in itself to understand and predict when and why this happens. …More decisions in the development of science and technology, but also in economy and politics are based on large-scale computations and simulations. Unfortunately, we can not take for granted that an honest error propagation analysis has been carried out to evaluate the results.

Table AI. The iteration of $X_t = 2X_{t-1}^2 - 1$ using Excel and Psion, using the same start conditions and a small error introduced

Number of iterations	IBM 486 using Excel spreadsheet		PSION 3a using Psion spreadsheet	
Start value	0.54321	0.54321000000001	0.54321	0.54321000000001
5	0.890035	0.890035	0.890035	0.890035
10	–0.84727	–0.84727	–0.84727	–0.84727
20	–0.07355	–0.07355	–0.07355	–0.07355
40	0.625099	0.614856	0.62497	0.614805
60	0.455086	–0.97999	–0.4463	–0.30702
80	–0.9822	–0.098716	0.306851	–0.80001
100	0.05050847	0.0349483	0.322846	–0.58814